Factors and Prime Numbers

WAGOLL: Highest Common Factor

hcf(84, 195)

Factors of 84 are

$$84 = 1 \times 84 = 2 \times 42 = 3 \times 28 = 4 \times 21 = 6 \times 14 = 7 \times 12$$

{1,2,3,4,6,7,12,14,21,28,42,84}

Factors of 195 are

$$195 = 1 \times 195 = 3 \times 65 = 5 \times 39 = 13 \times 15$$

{1,3,5,13,15,39,65,195}

(Common Factors are highlighted)

hcf(84,195) = 3

(The highest common factor is sometimes called the greatest common divisor.)

Prime Factors

All numbers greater than 1 can be made up as products of prime factors.

eg $45 = 9 \times 5 = 3 \times 3 \times 5 = 3^2 5^1$

Prime Numbers

Numbers are natural numbers with only two factors;

In Book IX of Elements, Euclid proved that there are an infinite number of primes;

2,3,5,7,11,13,17,19 are the prime numbers under 20;

If you have to think of a prime number, try numbers where the units column is 1,3,7 or 9 first;

Square numbers have an odd number of factors;

All other numbers have an even number of factors;

The opposite of a prime number is a composite number.

Eratosthenes Sieve

Write the numbers from 2 to 100. Go through the times tables from 2 to 10. Cross out the products which come up. The remaining numbers are prime numbers.

1 is not a prime number as it only has one factor.

Bertrand's Conjecture

For every integer, $n \geq 2$, there exists a prime p such that $n < p < 2n$.

This means that between any number (greater than 1) and double that number, there is a prime number: but is he right?

eg. Between 5 and 10, there is 7. Between 6 and 12, there is 7 and 11.

Prime Number Generator

$n^2 + n + 41$ = a prime number

Is this correct?

n	n^2	$n^2 + n + 41$	Prime number?
0	0	0+0+41=41	Yes
1	1	1+1+41=43	Yes
2	4	2+4+41=47	Yes

Goldback Conjecture

Every even number from 6 onwards can be written as the sum of two odd prime numbers.

Does this work up to 100?

WAGOLL: 2009 SATs Test

Write down a prime number that falls between 20 and 30.

Immediately, you should think of 21, 23, 27 and 29.

Then you should think:

21 = 7×3 so that is not prime;

27 = 3×9 so that is not prime.

Hence the answer could be 23 or 29.

By the end of this section of work, you should know:

- What is a prime number?
- What types of numbers are prime numbers?
- How can we investigate prime numbers?
- How can we search for patterns in prime numbers?

Did you know that whenever you order something from the Internet, your order is coded up using prime numbers?

People used to think number theory was a useless branch of mathematics. How wrong were they?

A mathematician's joke

Physicists use mathematical induction to prove that all odd numbers are prime.

1... well, no one can ever make their minds up.

3... yep. That is prime.

5... yes. That is certainly prime.

7... It is looking good for the odd number theory so far.

9... experimental error.

11... that is a prime number.